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CALIFORNIA UNIV LOS ANGELES SCHOOL OF ENGINEERING A--ETC F/6 20/11
COMPARISON OF THE BEST KNOWN FRACTURE CRITERIA WITH DATA ON NAT--ETC(U)
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UCLA-ENG-8040 NL

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COMPARISON OF THE BEST KNOWN FRACTURE CRITERIA
WITH DATA ON NATURALLY OCCURRING CRACKS.

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Interim report

by

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S.B. Batdorf

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The Department of the Navy
Office of Naval Research
Contract No. N0014-76-C-0445

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER U.C.L.A. - Eng. - 8040 ✓	2. GOVT ACCESSION NO. AD-A092480	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) COMPARISON OF THE BEST KNOWN FRACTURE CRITERIA WITH DATA ON NATURALLY OCCURRING CRACKS		5. TYPE OF REPORT & PERIOD COVERED Interim
7. AUTHOR(s) S.B. Batdorf		6. PERFORMING ORG. REPORT NUMBER
9. PERFORMING ORGANIZATION NAME AND ADDRESS School of Engineering and Applied Science, ✓ U.C.L.A., Los Angeles, 90024		8. CONTRACT OR GRANT NUMBER(s) N0014-76-C-0445
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research Attn.: Dr. N. Perrone Arlington, Virginia 22217		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Office of Naval Research - Branch Office 1030 E. Green Street Pasadena, California 91101		12. REPORT DATE July 1980
		13. NUMBER OF PAGES 13
		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Same as front page.		
<div style="border: 1px solid black; padding: 5px; text-align: center;"> DISTRIBUTION STATEMENT A Approved for public release: Distribution Unlimited </div>		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES Presented at the International Symposium on "Absorbed Specific Energy and/or Strain Energy Density," held September 17-19, 1980 in Budapest, Hungary.		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Fracture Mechanics Statistical Fracture Theory Mixed Mode Fracture Fracture Statistics Energy Density Theory		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Statistical fracture theory can be used to predict the statistics of fracture for equibiaxial tension when the statistics of fracture are known for simple tension. The result depends on the fracture criterion employed. It is shown that energy density theory (and therefore also specific energy theory) lead to better agreement with the experiment than any other well-known fracture criterion. ←		

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The concept of fracture as a result of crack instability was introduced by Griffith, who suggested two principles for determining the onset of catastrophic crack growth [1,2]. One was that fracture occurs when the strain energy released by crack extension is equal to the energy absorbed by the resulting free surfaces. The other was that fracture occurs when the tensile stress at the most highly loaded point on a crack surface reaches a critical value. The majority of criteria in use today for mixed mode fracture are related to one or the other of these two principles. One exception is a crude but convenient approximation which assumes that only the component of stress normal to the crack plane contributes to fracture. Others include the relatively recent energy density theory of Sih [3] and the equivalent specific energy theory of Gillemot [4].

A widely used experimental test of fracture criteria is to measure the strength and direction of extension of a known crack subjected to simple tension as a function of its orientation. Unfortunately, the predictions of several criteria are within the rather large experimental scatter, making a choice difficult.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100 101 102 103 104 105 106 107 108 109 110 111 112 113 114 115 116 117 118 119 120 121 122 123 124 125 126 127 128 129 130 131 132 133 134 135 136 137 138 139 140 141 142 143 144 145 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163 164 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183 184 185 186 187 188 189 190 191 192 193 194 195 196 197 198 199 200 201 202 203 204 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223 224 225 226 227 228 229 230 231 232 233 234 235 236 237 238 239 240 241 242 243 244 245 246 247 248 249 250 251 252 253 254 255 256 257 258 259 260 261 262 263 264 265 266 267 268 269 270 271 272 273 274 275 276 277 278 279 280 281 282 283 284 285 286 287 288 289 290 291 292 293 294 295 296 297 298 299 300 301 302 303 304 305 306 307 308 309 310 311 312 313 314 315 316 317 318 319 320 321 322 323 324 325 326 327 328 329 330 331 332 333 334 335 336 337 338 339 340 341 342 343 344 345 346 347 348 349 350 351 352 353 354 355 356 357 358 359 360 361 362 363 364 365 366 367 368 369 370 371 372 373 374 375 376 377 378 379 380 381 382 383 384 385 386 387 388 389 390 391 392 393 394 395 396 397 398 399 400 401 402 403 404 405 406 407 408 409 410 411 412 413 414 415 416 417 418 419 420 421 422 423 424 425 426 427 428 429 430 431 432 433 434 435 436 437 438 439 440 441 442 443 444 445 446 447 448 449 450 451 452 453 454 455 456 457 458 459 460 461 462 463 464 465 466 467 468 469 470 471 472 473 474 475 476 477 478 479 480 481 482 483 484 485 486 487 488 489 490 491 492 493 494 495 496 497 498 499 500 501 502 503 504 505 506 507 508 509 510 511 512 513 514 515 516 517 518 519 520 521 522 523 524
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Another type of experiment that can be used to compare fracture criteria employs unknown, naturally occurring cracks, and is statistical in nature. A major objective of statistical theories of fracture is to use the experimentally established fracture behavior in simple tension to predict fracture behavior in some other stress state. Since the result depends on the fracture criterion assumed [5], comparison of the prediction with experiment can be used to evaluate the relative merits of fracture criteria. A convenient second stress state for this purpose is equibiaxial tension. The purpose of the present paper is to carry out such a comparison.

THEORY

Assume that failure can occur due to any one of many independent and mutually exclusive mechanisms or causes, each having the infinitesimal probability of failure $(\Delta P_f)_i$. Under these circumstances, the probability of survival can be shown to be [5]

$$P_s = \exp[-\sum (\Delta P_f)_i] \quad (1)$$

In the case of a body containing Griffith cracks (open or closed elliptical cylinders) randomly oriented about axes normal to the stress plane and subjected to a uniform state of stress Σ

$$(\Delta P_f)_i = \left(\frac{\omega}{\pi} \right) \left(V \frac{dN(\sigma_c)}{d\sigma_c} \Delta\sigma_c \right) \quad (2)$$

Here V = volume

σ_c = critical stress, defined as the remote tensile stress that will cause fracture when applied normal to the crack plane.

$N(\sigma_c)$ = number of cracks per unit volume with critical stress $\leq \sigma_c$.

$\omega(\Sigma, \sigma_c)$ = angle within which normal to crack plane must lie to cause fracture.

The right hand side of (2) is simply the probability that a crack in the critical stress range $\Delta\sigma_c$ is present times the fraction of such cracks with an orientation that will result in fracture. Inserting (2) into (1) and going to the limit of small $\Delta\sigma_c$

$$P_s(\Sigma) = \exp \left[-V \int_0^\infty \frac{\omega}{\pi} \frac{dN}{d\sigma_c} d\sigma_c \right] \quad (3)$$

We note that the integral in (3) is finite because $\omega = 0$ when $\sigma_c > \sigma_1$, the maximum principal stress (in some cases, to be discussed later, σ_c may exceed σ_1 by several percent for ω to be zero).

In the case of equibiaxial tension

$$\omega = \pi \quad (\sigma_c < \sigma) \quad (4a)$$

$$= 0 \quad (\sigma_c > \sigma) \quad (4b)$$

As a result, (3) reduces to the simple form

$$P_s(\sigma, \sigma) = \exp[-VN(\sigma)] \quad (5)$$

To facilitate the comparison between uniaxial and equibiaxial fracture behavior, we assume with Weibull [6] that N takes the form

$$N(\sigma_c) = k\sigma_c^m \quad (6)$$

Then

$$P_s(\sigma, \sigma) = \exp[-Vk\sigma^m] \quad (7)$$

$$P_s(\sigma, 0) = \exp\left[-Vkm \int_0^\infty \left(\frac{\omega}{\pi}\right) \sigma_c^{m-1} d\sigma_c\right] = \exp[-Vk'\sigma^m] \quad (8)$$

where

$$k' = km \int_0^\infty \frac{\omega}{\pi} x^{m-1} dx \quad (9)$$

$$x \equiv \sigma_c / \sigma \quad (10)$$

The desired relation between uniaxial and equibiaxial fracture statistics can thus be expressed in the form

$$\frac{\ln P_s(\sigma, \sigma)}{\ln P_s(\sigma, 0)} = \frac{k}{k'} = \left[m \int_0^\infty \frac{\omega}{\pi} x^{m-1} dx \right]^{-1} \quad (11)$$

As a simple illustration of the use of (11), consider the case of shear-insensitive cracks. With such cracks, fracture depends only on the normal component of stress σ_n which for simple tension is given by

$$\sigma_n = \sigma \sin^2 \beta \quad (12)$$

Here β is the angle between tensile axis and crack plane. The critical angle β_c occurs when $\sigma_n = \sigma_c$, or

$$\beta_c = \sin^{-1} \sqrt{\frac{\sigma_c}{\sigma}} \quad (13)$$

as shown in Fig. 1a.

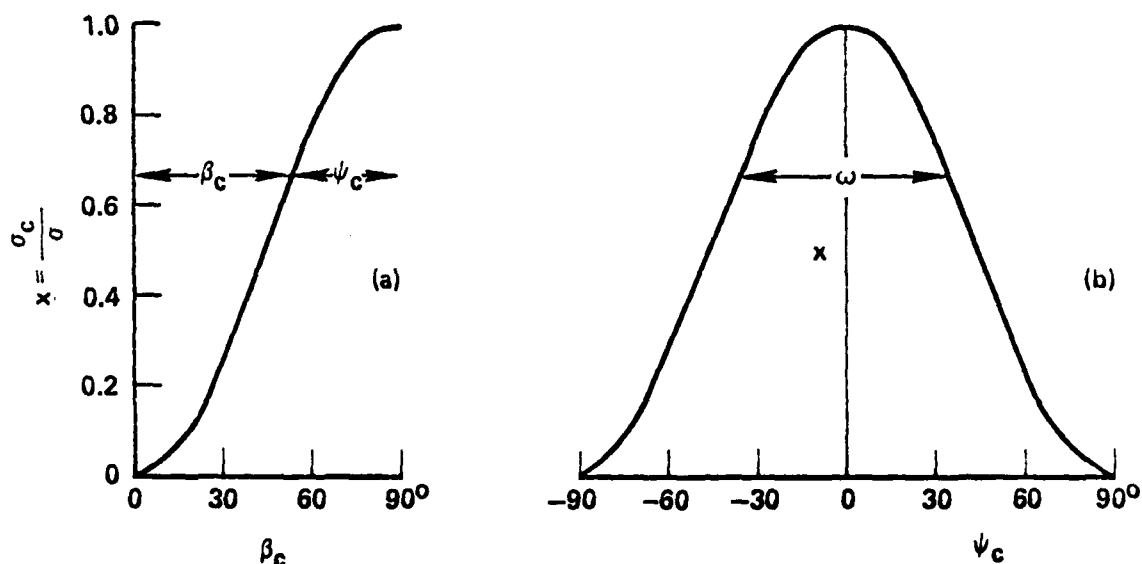


Figure 1. Critical angle and ω for shear-insensitive cracks.

The angle ω is $2\psi_c$ where for the case under consideration

$$\psi_c = \cos^{-1} \sqrt{\frac{\sigma_c}{\sigma}} \quad (14)$$

as is evident from inspection of Fig. 1b. Inserting these relations in (11), we obtain

$$\frac{k}{k'} = \left[\frac{2}{\pi} \int_0^1 \cos^{-1} \sqrt{x} \, x^{m-1} \, dx \right]^{-1} = F_1(m) \quad (15)$$

The relation between k/k' and m is shown as the top curve in Fig. 2. This curve gives the results of Weibull theory, which tacitly assumes that cracks are shear-insensitive [7].

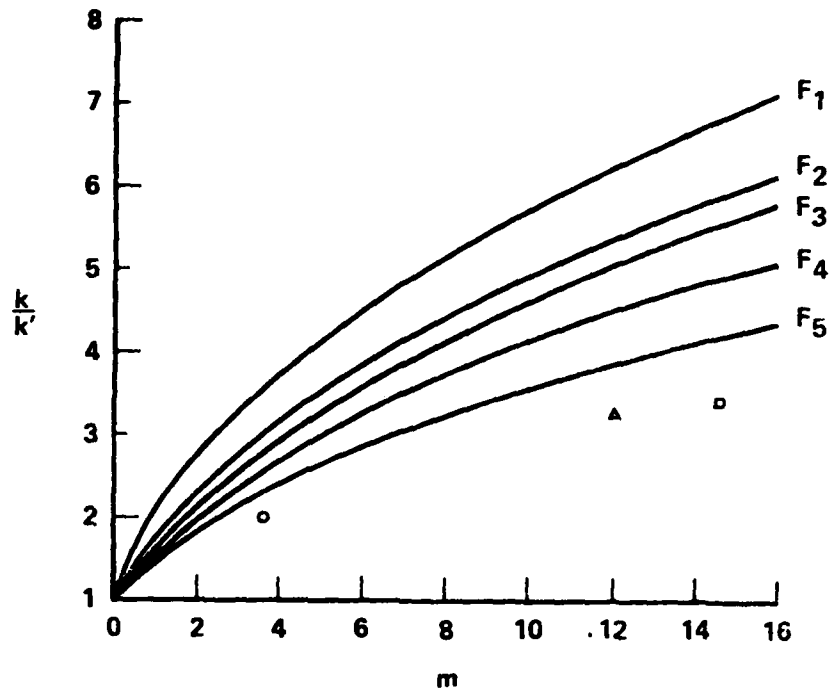


Figure 2. Uniaxial/equibiaxial relation for five fracture criteria.

Other fracture criteria leading to other functional relations between ω and x and therefore other functions $F_1(m)$ have been discussed in [5] and [8]. For the sake of brevity, we content ourselves here with the identification of crack types and fracture criteria leading to curves F_2 to F_5 , as shown in Table I.

CURVES	CRACK TYPE	FRACTURE CRITERION
$F_2(m)$	Open; Griffith	Tensile stress in crack surface.
$F_3(m)$	Open; Penny-shaped	
$F_4(m)$	Open or closed; Griffith	Energy release rate for coplanar extension.
$F_5(m)$	Open or closed; Penny-shaped	

Table I. Identification of curves in Fig. 2.

The curves for F_3 and F_5 depend on Poisson's ratio ν , which was assumed to be 0.25.

In addition to F_1 to F_5 , Fig. 2 shows experimental results for three different materials. The circle represents test results taken on tubes of pyrex glass [9]. The triangle represents test results on ATJS graphite bars and disks spun to failure [10]. In the case of the disks, only the failures originating very near the center were included in the analysis. The square represents results on beams and disks of alumina subjected to bending stress [11]. The test results are summarized in Table I.

Material	m	k/k'
Pyrex	3.35	2.0
ATJS Graphite	12.00	3.25
Alumina	14.3	3.35

Table II. Experimental k/k' relation.

It is evident that agreement with the experiment leaves something to be desired. We next turn to predictions based on energy density theory and of fracture criteria strain energy release rate for non-coplanar crack extension.

Sih [12] has shown that for an inclined Griffith crack in simple tension, the critical energy density function is given by

$$S_c = a_{11}k_1^2 + 2a_{12}k_1k_2 + a_{22}k_2^2 \quad (16)$$

where

$$a_{11} = \frac{1}{16\mu} [3-4\nu-\cos\theta_0](1+\cos\theta_0) \quad (17a)$$

$$a_{12} = \frac{1}{16\mu} 2\sin\theta_0 [\cos\theta_0 - (1-2\nu)] \quad (17b)$$

$$a_{22} = \frac{1}{16\mu} [4(1-\nu)(1-\cos\theta_0) + (1+\cos\theta_0)(3\cos\theta_0-1)] \quad (17c)$$

and where μ is the shear modulus. For Griffith cracks

$$k_1 = \sigma a^{1/2} \sin^2\beta \quad (18a)$$

$$k_2 = \sigma a^{1/2} \sin\beta \cos\beta \quad (18b)$$

while for penny-shaped cracks

$$k_1 = \frac{2}{\pi} \sigma a^{1/2} \sin^2\beta \quad (19a)$$

$$k_2 = \frac{4}{\pi} \sigma a^{1/2} \frac{\sin\beta \cos\beta}{2-\nu} \quad (19b)$$

θ_0 is chosen by minimizing S with respect to θ .

A plot of the resulting $x(\beta)$ is shown in Fig. 3. Also shown is a plot of the corresponding relation for penny-shaped cracks.

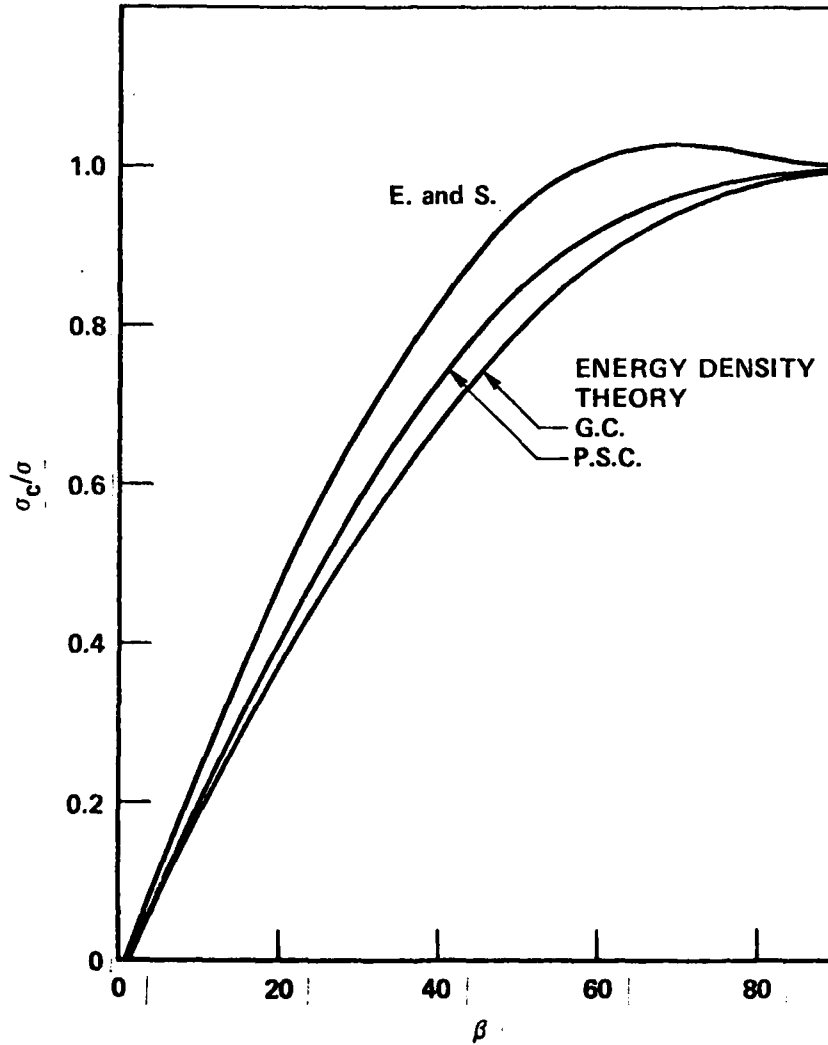


Figure 3. Critical angles for selected fracture criteria.

A major motivation underlying Sih's early efforts in developing energy density theory was the fact that up to that time, mathematical difficulties had precluded accurate calculation of strain energy release rates for non-coplanar crack extension. Recently two solutions have been offered for the case of closed Griffith cracks, one by Palaniswamy and Knauss [14], and one by Wu [15]. The two solutions are in excellent agreement with each other, and lead to virtually the same strength for an angled crack as that obtained from the theory proposed by Erdogan and Sih [16] using Griffith's stress criterion. This strength can be readily calculated from the following relation [17]

$$\frac{\sigma_c}{\sigma} = \cos \frac{\theta_0}{2} \sin \beta \left(\sin \beta \cos^2 \frac{\theta_0}{2} - \frac{3}{2} \cos \beta \sin \theta_0 \right) \quad (19)$$

where θ_0 is found from the equation

$$\tan\beta \sin\theta_0 + 3\cos\theta_0 = 1. \quad (20)$$

The resulting function $x(\beta) = \sigma_c/\sigma(\beta)$ is shown as the curve in Fig. 3, labeled "E. and S." Note that this curve reaches a maximum value of about 1.03 at $\beta_c \sim 70^\circ$, whereas all the others considered here reach a maximum value of $x = 1$ at $\beta = 90^\circ$. This peculiarity has important consequences for the corresponding value of k/k' .

When the relations of Fig. 3 are used to evaluate ω and the results substituted in (11), the curves for k/k' shown in Fig. 4 are obtained. The energy density criterion leads to fairly good agreement with the experiment in the case of Griffith cracks, and quite good agreement for penny-shaped cracks. The agreement of statistical theory based on the use of available theories for non-coplanar energy release rate is so poor as to be (in the opinion of the present author) unacceptable because it leads to an incorrect variation of k/k' with m .

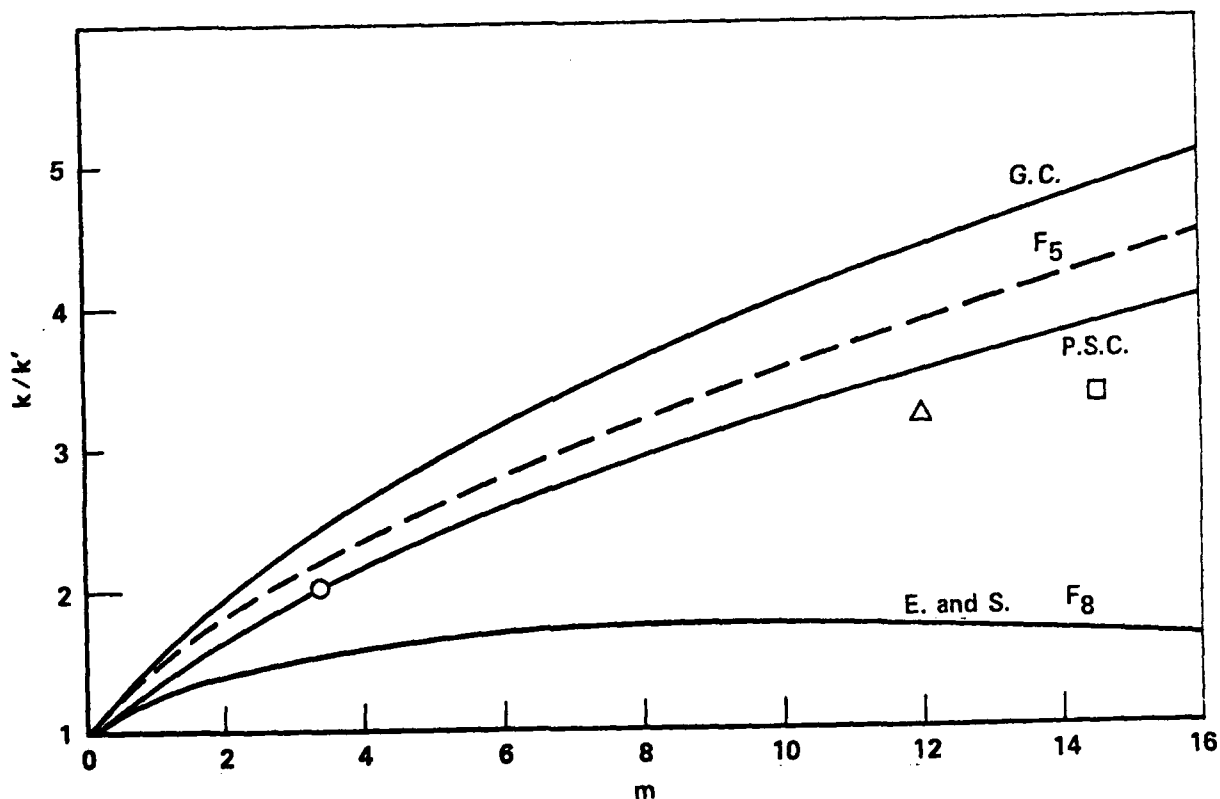


Figure 4. Uniaxial/equibiaxial relation for selected fracture criteria.

DISCUSSION

The preceding analysis considers open or closed, Griffith or penny-shaped cracks, fracturing according to any one of nine different fracture criteria (F_8 does double duty, representing both the σ_0 and the non-coplanar energy release rate criteria). Best agreement with the experiment was obtained using energy density theory as applied to closed penny-shaped cracks. The poor agreement of the relation F_8 with the experiment is quite surprising since it appears to offer in principle an accurate mathematical solution of a physically valid principle (energy balance for non-coplanar crack extension). The low value for the ratio k/k' can be traced to the influences on the integral in (9) of the portion of the $x(\beta)$ curve that is greater than unity. This is not only physically implausible, but leads to the incredible prediction that as $m \rightarrow \infty$, $k/k' \rightarrow 0$.

It was to be expected that of the two energy density solutions, the one based on use of penny-shaped cracks is in best agreement with the experiment since such cracks are more closely related to naturally occurring cracks than Griffith cracks. To validate the conclusion that it agrees with uniaxial/biaxial strength data better than any other well-known theory, however, it is necessary to justify the use of two-dimensional statistical theory (eq. (11)) rather than the three-dimensional theory that might appear more appropriate [5]. This justification lies in the surprising fact that the two theories lead to the same k/k' ratio. This was proved in [16] for the relations F_1 and F_4 (see Fig. 2), and has subsequently been shown by the present author to be a generally valid result.

It should be pointed out that the present paper is not the first to discuss statistical fracture theory using energy density theory as the fracture criterion. In a couple of papers published in 1977 [19, 20] Jayatilaka and Trustrum considered uniaxial and biaxial stress states, respectively. However, their results are flawed by the use of a crack size distribution, and therefore a functional relation for $N(\sigma_c)$, that was determined experimentally for surface cracks in glass. There is no reason to expect that the same law should hold for interior cracks in polycrystals. In fact, it is very likely it would not hold even for another sample of the same glass, since surface flaws in glass are generally considered to be due to handling damage. In addition, no comparison of their results were made with experimental data, or even with predictions of statistical fracture behavior based upon other fracture criteria.

Finally, there is the question of the reliability of the data. The fact that the three data points (a) each represent many tests, (b) involve different types of experiments, (c) were carried out by different experimenters on different materials, and (d) collectively establish a trend closely related to what would be expected on theoretical grounds, tends to establish confidence in their validity. It would nevertheless be highly desirable to obtain additional experimental data to give greater assurance to the conclusions reached in this investigation.

CONCLUSIONS

1. The relation between the fracture statistical behavior of specimens in simple tension and that in equibiaxial tension depends on both the assumed crack shape and the fracture criterion employed.
2. The present study investigates the uniaxial/equibiaxial relation for Griffith and penny-shaped cracks for a number of fracture criteria. Best (in fact, very good) agreement with the experiment was obtained assuming energy density (or absorbed specific energy) theory and penny-shaped cracks.
3. Contrary to what might be expected, the maximum energy release rate criterion for non-coplanar extension of Griffith cracks leads to a uniaxial/equibiaxial fracture relation that is in poor agreement with experiment.

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